MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

14. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University Post Office.

"The center of a sphere of radius c moves in a circle of radius a and generates thereby a solid ring, as an anchor ring: prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to a plane is $\dfrac{\pi^2\rho ac^2}{4}(4a^2+3c^2)$."

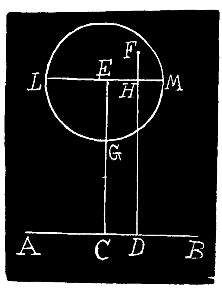
IV. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let E be the centre of the sphere, AB, the axis of revolution, be the axis of x, F any point in area of the circle, $DF=y$, $HF=y'$, $CE=a$, $GE=c$.

Let dA be the element of area, then the volume of the elementary ring generated by dA is $2\pi ydA$, and its mass, $2\pi\rho ydA$.

$\therefore$ the moment of inertia of this elementary ring relative to the axis of x, is $2\pi\rho y^3dA$.

$\therefore$ the moment of inertia required $=I=2\pi\rho\Sigma y^3dA$.



$I=2\pi\rho\Sigma(a+y')^3dA$, since $y=a+y'$,

$$=2\pi\rho\Sigma(a^3+3a^2y'+3ay'^2+y'^3)dA.$$

But the curve is symmetrical with respect to the axis LM. $\therefore$ $\Sigma y'dA=0$, $\Sigma y'^3dA=0$, and by definition, $\Sigma y'^2dA=Ak^2=\pi c^2k^2$. $\therefore$ $I=2\pi\rho a\times\pi c^2(a^2+3k^2)$

$=2\pi^2\rho ac^2(a^2+3k^2)$; but $k^2=$ radius of gyration $=\dfrac{c^2}{4}$.

$$\therefore\quad I=2\pi^2\rho ac^2(a^2+\tfrac{3}{4}c^2)=\frac{\pi^2\rho ac^2}{2}(4a^2+3c^2).$$

V. Solution by P. H. PHILBRICK, M. Sc., C. E., Lake Charles, Louisiana.

$OP=a=$ the radius of directing circle and $Pa=c=$ radius of the sphere.

Take tangent Px for the axis of x. Let $aPx=\theta$, $aPc=d\theta$. Draw abR and cdN perpendicular to Px. Take m at middle of ab. Now $am=c\cos\theta$, $\therefore$ $aR=a+c\cos\theta$ and $bR=a-c\cos\theta$. Thickness of $abcd=c\cos\theta d\theta$.

Area of circle, radius aR, $=\pi(a+c\cos\theta)^2$.

$\therefore$ vol. of circular lamina $aR=\pi(a+c\cos\theta)^2\, c\cos\theta d\theta$ and moment of Inertia about axis through O perpendicular to plane

$$OPx=I=\frac{\pi}{2}(a+c\cos\theta)^4\, c\cos\theta d\theta.$$

Similarly, moment of Inertia of lamina $bR=I'$

$$=\frac{\pi}{2}(a-c\cos\theta)^4\, c\cos\theta d\theta.$$

$\therefore$ moment of Inertia of lamina $ab=I-I'=I''=4\pi ac^2(a^2\cos^2\theta d\theta+c^2\cos^4\theta d\theta)$.

$$\text{But}\int_0^\pi\cos^2\theta d\theta=\frac{\pi}{2};\ \text{and}\int_o^\pi\cos^4\theta d\theta=\tfrac{3}{4}\int_o^\pi\cos^2\theta d\theta=\frac{3\pi}{8}.$$

$\therefore$ $I''=\dfrac{\pi^2ac^2}{4}(8a^2+6c^2)^4$. The moment of Inertia about an axis through O and in the plane OPx is of course one half of the above amount, or $\dfrac{\pi^2ac^2}{4}(4a^2+3c^2)$.

[NOTE.—An excellent solution of this problem was received from Professor Wiggins, Richmond, Indiana, but as it has been lost we are unable to publish it. —Editor.]

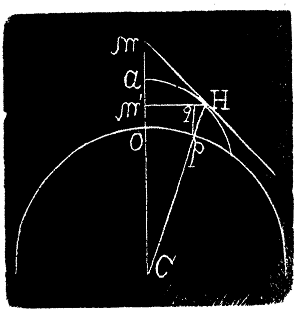
15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Show that the *eastward deviation* of bodies falling from a great height is

$$E_d=\frac{4\pi t(H-\frac{1}{2}\triangle)\cos\phi}{3T}.$$

Solution by the PROPOSER.

Let r=the radius of the earth, T=86164.2=the *mean-time* seconds in a siderial day, ϕ=the observer's latitude, $(r+H)$=the distance from the center of the earth to the point from which the body fell, and t=the time of the body's motion in seconds; then the *horizontal velocity* of the falling body in the direction of the tangent to the circle of latitude at the place of observation, is



$V=\dfrac{2\pi(r+H)\cos\phi}{T}$ ....(1); and this velocity is caused by the rotation of the earth. Obviously the *horizontal space* described by the body in the time t, is $S_1=Vt=\dfrac{2\pi t(r+H)\cos\phi}{T}$ ....(2).

The point on the earth and vertically below the point from which the body fell, passes during this time t through an arc

$$S_2=\frac{2\pi tr\cos\phi}{T}\ldots.(3).$$